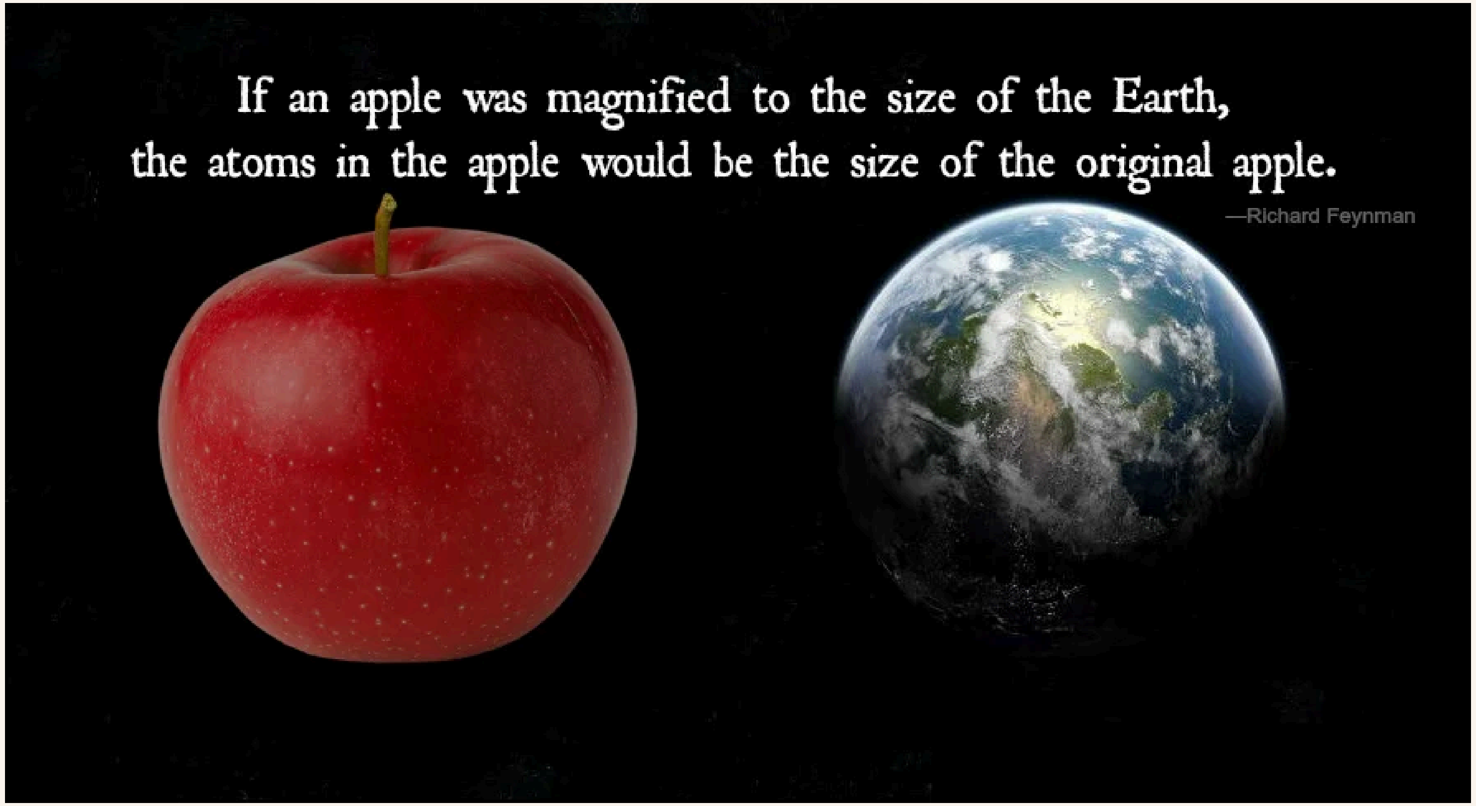
Indices



* the Calcium and Potassium atoms

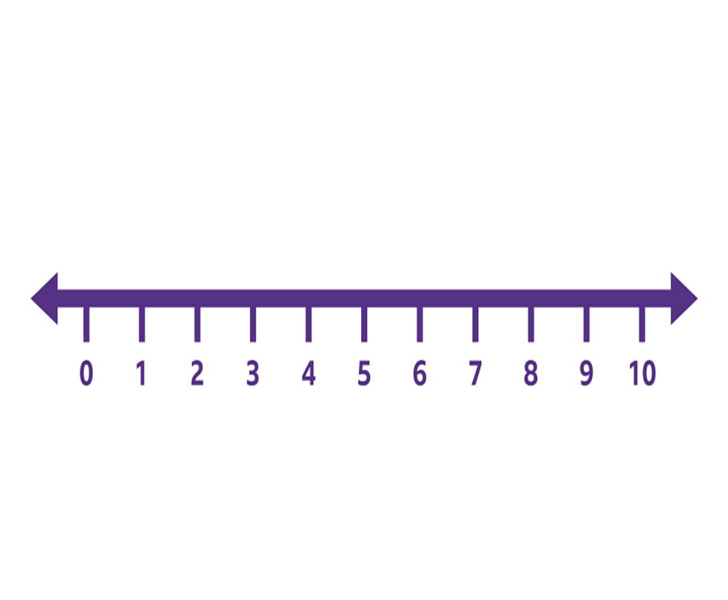
Learning Intention

Topic: Index Notation

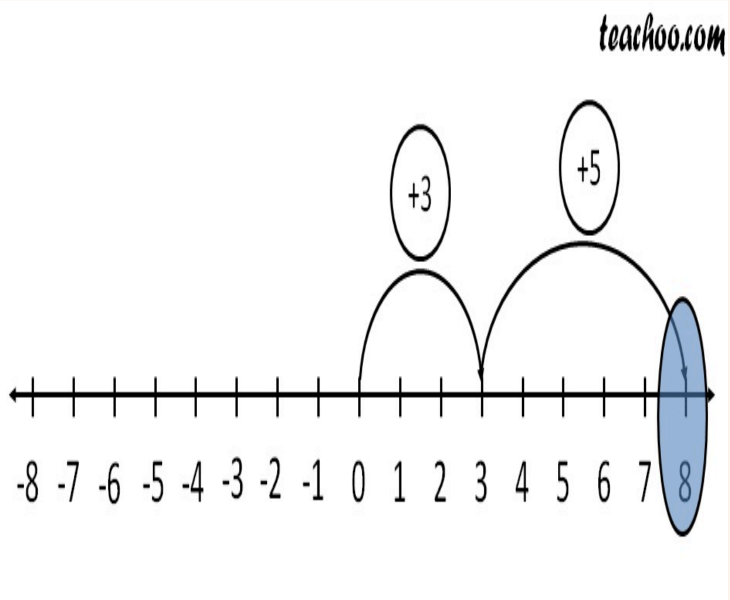
7A	**1 I can write expressions in expanded form and simplify.** e.g. Write the following in expanded form and simplify. **a** $(2ab)^3$ **b** $\left(\frac{2}{7}\right)^2$	☐
7A	**2 I can write numbers and expressions using index form.** e.g. Write the following in index form: **a** $\frac{7}{9} \times \frac{7}{9} \times \frac{7}{9} \times \frac{7}{9}$ **b** $2 \times a \times a \times b \times a \times b$	☐
7A	**3 I can express a number as a product of its prime factors.** e.g. Express 92 as a product of its prime factors.	☐
7A	**4 I can evaluate expressions involving indices using substitution.** e.g. If $a = 2$, $b = -3$ and $c = 11$, evaluate the following. **a** $(ab)^3$ **b** $\left(\frac{b}{\ }\right)^2$	☐

Where we've come from

Counting

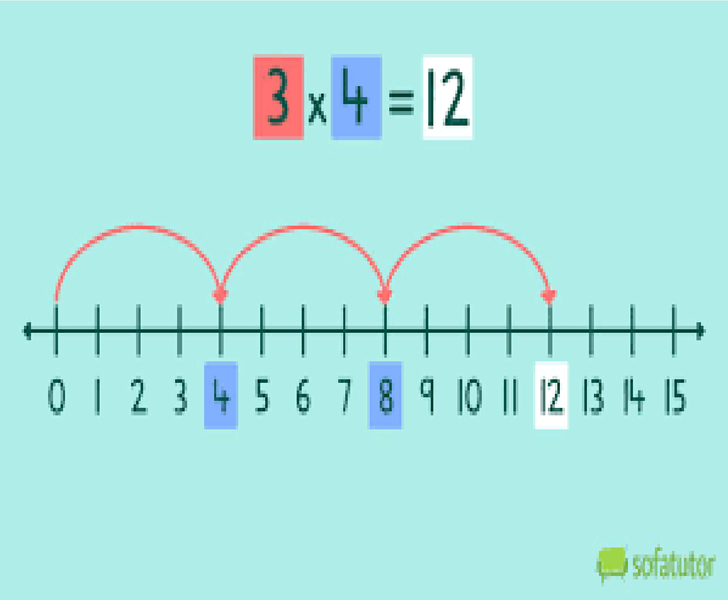


Addition



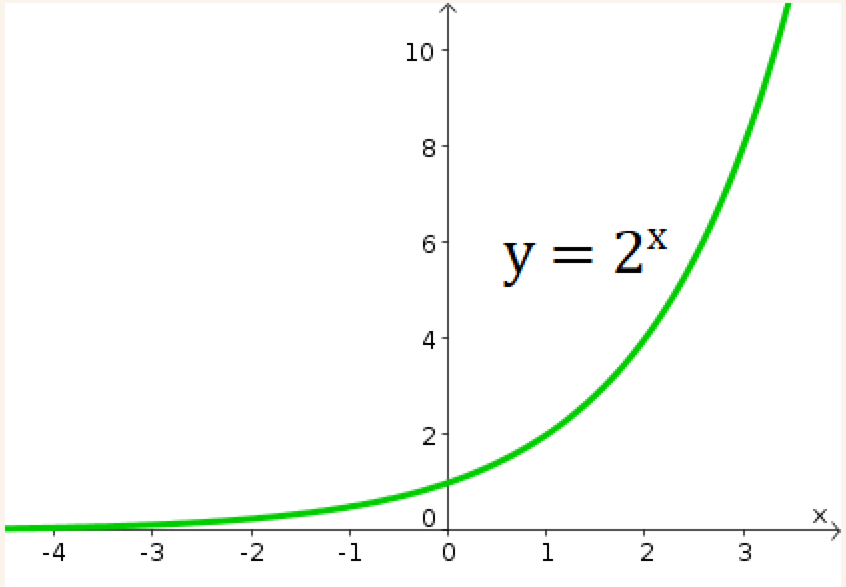
Jumps in Counting

Multiplication

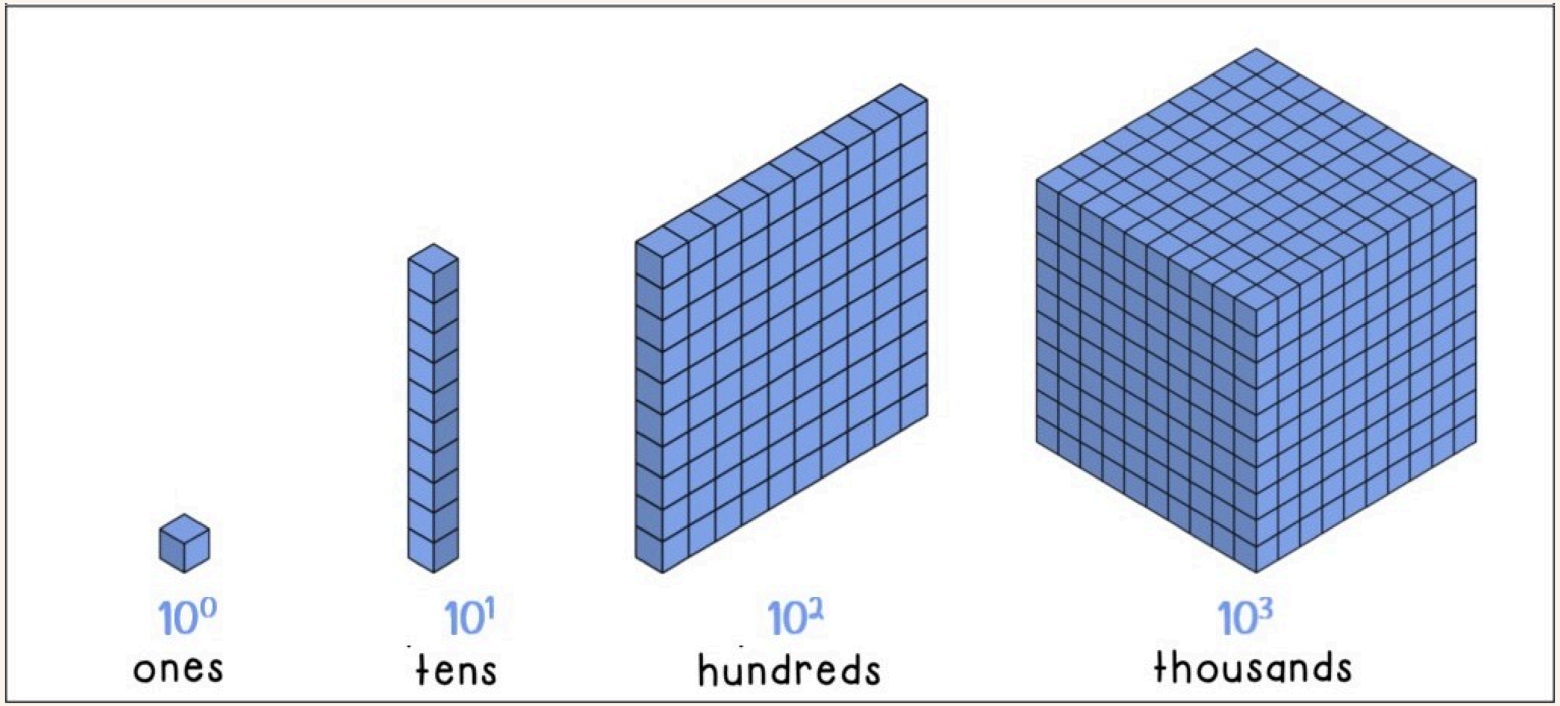


Repeated addition

Where We're Going We Don't Need Number Lines

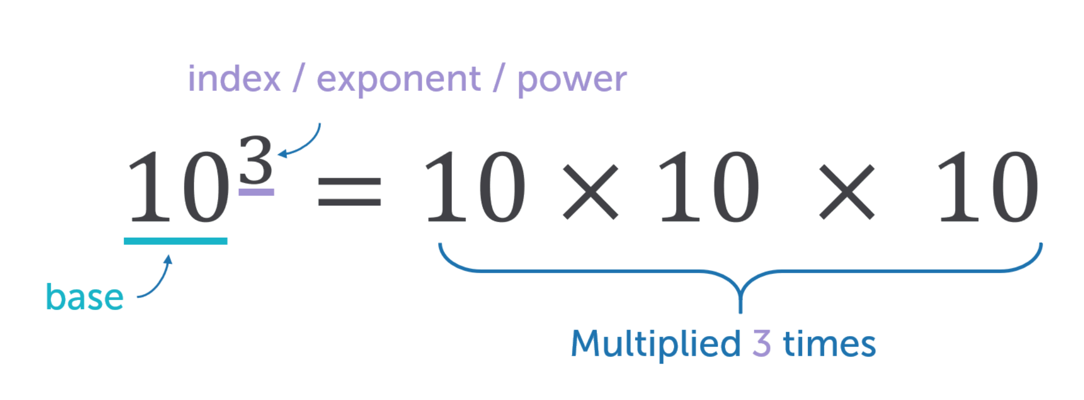


Where We're Going: Indices!



- Repeated multiplication
- Like place value, but for numbers other than just 10
- I promise we'll stop stacking operations after indices

Notation



- We would pronounce this as "10 to the **power** of 3"
- What would $x \times x \times x \times x$ be?
  - x^4
- Note: 3^2 does *not* mean 3×2, it means 3×3

Examples

- What would $6 \times 6 \times 6$ be in index notation?
- Evaluate: 6^3 (What does **evaluate** mean?)

What's 5^1?

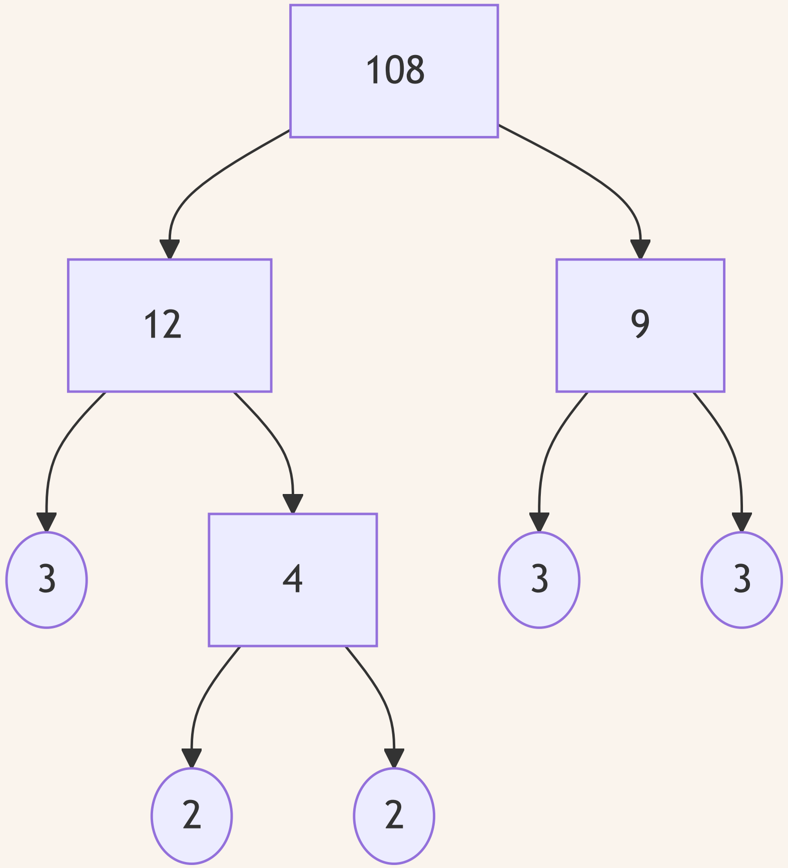
- 5^2 is 5 multiplied by itself once, or 5×5
- What's 5 once, if we don't multiply it with another 5?
  - Just 5 ‼️

$a^1 = a$

***for literally every single number out there**

- 1 or 0 as bases are special: any power we raise them to, they stay the same
- i.e. $1^n = 1$, $0^n = 0$

Prime Factor Form and Index Form



- You might remember drawing factor trees
    - Remember, a prime has only itself and 1 as factors
- This is the factor tree of 108
- From this, we can identify that 108's prime factors
- We call $2 \times 2 \times 3 \times 3 \times 3$ the **prime factor form** of 108
- We can express this as $108 = 2^2 \times 3^3$
- We call this the **index form** of 108

Repeated division method

2	48
2	24
2	12
2	6
3	3
	1

$$48 = 2 \times 2 \times 2 \times 2 \times 3$$
$$= 2^4 \times 3$$

Repeated Division

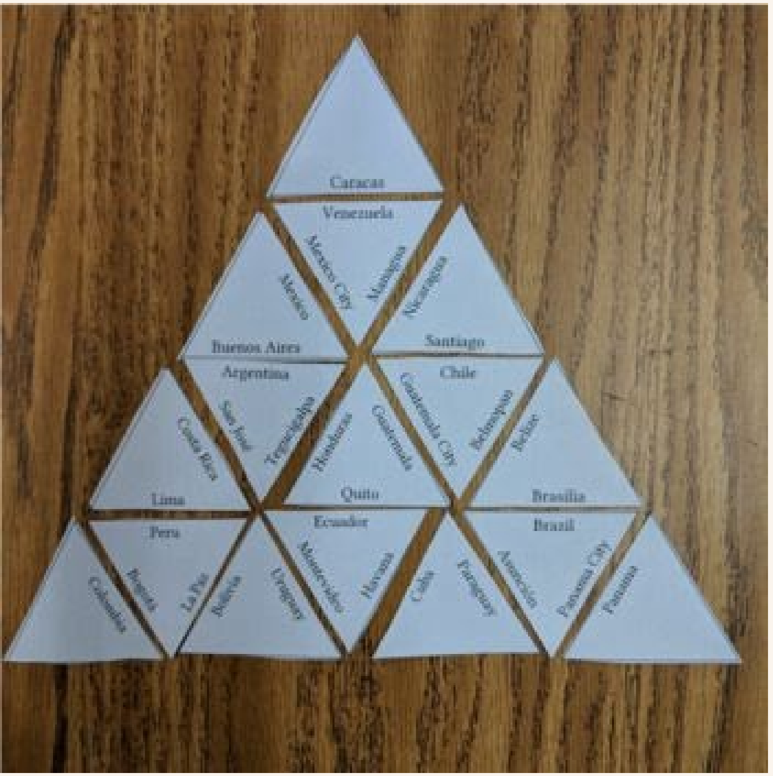
- We can also find the factors of a number by **repeated division**
- We divide by prime factors, until we reach 1
- We may need to use bigger primes but start with the small ones like 2 and 3

Examples

- Find the index form of 120 by drawing the factor tree

Your Turn

- Find the index form of 84 by drawing the factor tree



Now: Tarsia Activity

- Each column in the class has a set of triangles
- Each triangle edge has a question or answer on it
- Find the edge pairs where the question and answer match to assemble the big triangle
  (the blank sides are the outer edge)
- Work with your friends and paste the assembled tarsia on the A3 piece of paper

Learning Intention

Topic: Index Laws for Multiplication and Division

7B	**5 I can simplify expressions with numerical bases using index laws.** e.g. Simplify, giving your answer in index form. **a** $4^3 \times 4^4$ **b** $7^5 \div 7$	☐
7B	**6 I can use the index law for multiplication.** e.g. Simplify the following. **a** $a^3 \times a^7$ **b** $9x^2 \times 3x^3$	☐
7B	**7 I can use the index law for division.** e.g. Simplify $y^6 \div y^2$.	☐
7B	**8 I can combine index laws for multiplication and division.** e.g. Simplify: $\frac{3a^2b \times 4ab^3}{8a^2b^3}$.	☐

Index Laws: Multiplication

What happens if we multiply $2^5 \times 2^3$?

$$2^5 = 2 \times 2 \times 2 \times 2 \times 2$$

= 32

$$2^3 = 2 \times 2 \times 2$$

= 8

$$2^5 \times 2^3 = 32 \times 8$$

= 256

Here's the cool part

$$2^8 = 256$$

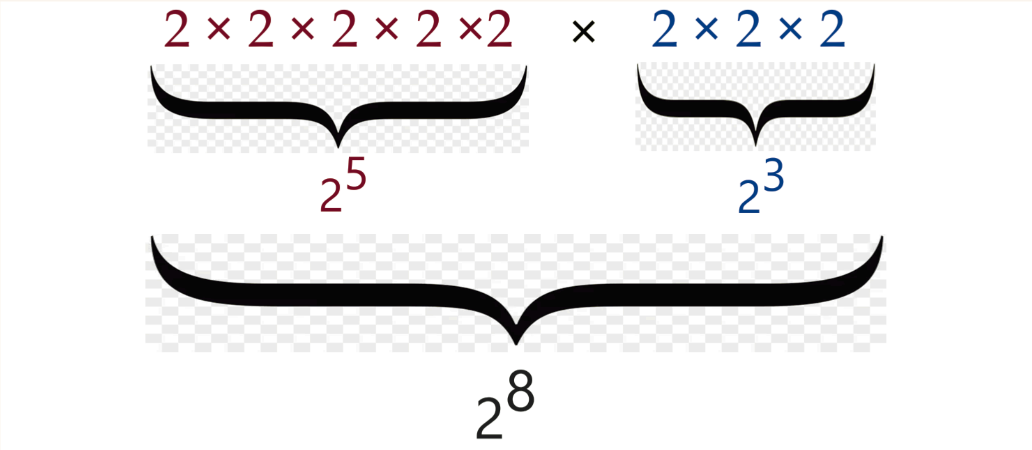
And: 8 = 5 + 3

Index Law: Multiplication

$$a^m \times a^n = a^{m+n}$$

Let's see why this works: Back to $2^5 \times 2^3$

- 2^5 is 2 multiplied 5 times
- 2^3 is 2 multiplied 3 times
- So when we multiply $2^5 \times 2^3$, we have $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
  - which is 2 multiplied 8 times!



Index Law: Division

So that's multiplication, what about division?

Let's try to figure it out by comparing to multiplication

- What is $3^5 \div 3^2$?
- $3^5 = 3 \times 3 \times 3 \times 3 \times 3$

  = 243
- $3^2 = 3 \times 3$

  = 9

- So $3^5 \div 3^2$ = 243 ÷ 9 = 27
  And $27 = 3^3$
- How do we go from 5 to 3?
  - Subtract 2
  - Does this have anything to do with the fact that we divided by $3^{\mathbf{2}}$ (3 to the power of **2**)?
- Yes!
- $$\boxed{a^m \div a^n = a^{m-n}}$$

Learning Intention

Topic: Power of Powers and Zero Index

7C	**9 I can simplify expressions containing the zero index.** e.g. Evaluate using the zero index: $3^0 + 3a^0$	☐
7C	**10 I can use the index law for power of a power.** e.g. Simplify $5(x^3)^6$	☐
7C	**11 I can combine index laws.** e.g. Simplify the following. **a** $(x^2)^3 \times (x^4)^2 \div x^{14}$ **b** $\frac{7a^3 \times 2a^4}{4a^5}$	☐

Powers of Powers

- What happens if we raise a number in index form to another power?
- For example, what would $(a^2)^5$ look like?

$$
\begin{aligned}
(a^2)^5 &= \underbrace{a^2}_{} \times \underbrace{a^2}_{} \times \underbrace{a^2}_{} \times \underbrace{a^2}_{} \times \underbrace{a^2}_{} \\
&= a \times a \times a \times a \times a \times a \times a \times a \times a \times a \\
&= a^{10}
\end{aligned}
$$

- So, all we've done is multiply the two indices
- In other words, the power of a power is just another power (see, I said we wouldn't need to stack operations past indices)

Zero index

What's 2^0?

- 2^3 = 8
- 2^2 = 4
- We said 2^1 is 2
- So we divide by 2 at each step
- Now, let's continue the pattern: what's 2^0?
  - $\frac{2}{2} = 1$ 🎉

$a^0 = 1$

***for every number except 0**

- Start at the center (x^8) and move outward by multiplying your current value by the new cell
- Example: If I move up from x^8, I multiply with $\frac{1}{x^{10}}$ and get $\frac{1}{x^2}$
- You win if you can reach the edge with the result 1

1 = WIN

$\frac{1}{x^5}$	x^3	x^{10}	x^{-8}	x^2
x^1	x^4	$\frac{1}{x^{10}}$	x^6	x^{-1}
x^{-6}	x^{-3}	x^8	$\frac{1}{x^2}$	x^9
$\frac{1}{x^{-5}}$	x^7	$\frac{1}{x^7}$	x^{-9}	x^0
x^{-4}	$\frac{1}{x}$	x^0	x^3	x^4

1 = WIN

Learning Intention

Topic: Index Laws Extended

7D

12 I can use index laws to rewrite expressions.

e.g. Simplify, using index laws.

a $(2a^4)^3$ **b** $\left(\frac{-2b^2}{a^3}\right)^4$

☐

7D

13 I can combine index laws to simplify expressions.

e.g. Simplify the following.

a $x(-xy^2)^3$ **b** $\left(\frac{a^2b^3}{2}\right)^2 \times \frac{4}{(ab)^2}$

☐

Index Laws Expanded

What would $(3 \times 4)^2$ be?

- First let's evaluate what's in our brackets
- We have $(3 \times 4)^2 = 12^2$
- = 144
- Meanwhile: $3^2 \times 4^2 = 9 \times 16$
- = 144
- So $(3 \times 4)^2 = 3^2 \times 4^2$????

This is how it works:

Let's take any two numbers a and b

- Consider: $(a \times b)^6$
- $$(a \times b)^6 = \overbrace{ab \times ab \times ab \times ab \times ab \times ab}^{\text{6 factors of } ab}$$
  $$= \overbrace{a \times a \times a \times a \times a \times a}^{\text{6 factors of } a} \times \overbrace{b \times b \times b \times b \times b \times b}^{\text{6 factors of } b}$$
  $$= a^6 \times b^6$$
- $(a \times b)^n = a^n \times b^n$ **for any** a, b, n

Same way:

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$\left(\frac{a}{b}\right)^6 = \overbrace{\frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b}}^{6 \text{ factors of } \frac{a}{b}}$$

$$= \frac{\overbrace{a \times a \times a \times a \times a \times a}^{6 \text{ factors of } a}}{\underbrace{b \times b \times b \times b \times b \times b}_{6 \text{ factors of } b}}$$

$$= \frac{a^6}{b^6}$$

Common Mistakes

- $a^m \times b^n \neq (a \times b)^{m+n}$ or $(a \times b)^m$
- e.g. $4^3 \times 2^2$
  - They aren't the same base so we can't apply the rule (yet)
  - $4 = 2^2$ so we can express 4^3 with 2 as a base
  - We apply the Powers of Powers rule: $4^3 = (2^2)^3$

    $= 2^{3\times 2}$

    $= 2^6$
  - Now our original problem looks like: $2^6 \times 2^2$
  - And we can apply our index multiplication rule!
  - $4^3 \times 2^2 = 2^6 \times 2^2$

    $= 2^{6+2} = 2^8$

The same thing goes for division

- $a^m \div b^n \neq (a \div b)^{m+n}$ or $(a \div b)^m$
- e.g. $25^5 \div 5^3$
  - Again, we need to express 25 as 5^2
  - We apply the Powers of Powers rule: $25^2 = (5^2)^5$
    $= 5^{2\times 5}$
    $= 5^{10}$
  - Now our original problem looks like: $5^{10} \div 5^3$
  - And we can apply our index division rule!
  - $25^5 \div 5^3 = 5^{10} \div 5^3$
    $= 5^{10-3} = 5^7$

Learning Intention

Topic: Negative Indices

7E	**14 I can express negative indices in positive index form.** e.g. Rewrite the following with positive indices only: **a** $4x^{-2}$ **b** $\frac{5}{2^{-3}}$	☐
7E	**15 I can evaluate expressions involving negative indices.** e.g. Write the following with a positive index and then as a fraction. **a** 5×2^{-3} **b** 4×10^{-2}	☐

Do negative indices exist?

- Yes they do! What do they mean?
- Well, negative integers are the opposite of positive integers
- What's the opposite of multiplication?
  - **Division**
- So, exponents with a negative index are the notation for repeated division!
- $a^{-n} = \frac{1}{a}$ **multiplied** n **times**
- (This is because we can express division as a multiplication of a fraction, e.g. 5 ÷ 4 = $5 \times \frac{1}{4}$)

What about $\frac{1}{a^{-m}}$?

- So we know $a^{-n} = \frac{1}{a^n}$
- How do we **flip** $\frac{1}{a^{-m}}$?
- $\frac{1}{a^{-m}} = a^m$
- So we say for example: $5^{-2} = \frac{1}{5^2}$ and $\frac{1}{3^{-4}} = 3^4$
- Basically: A number with a negative index is "unhappy" where it is and wants to be flipped
- The **reciprocal** of a fraction is the fraction "flipped upside down"
- **The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$**

Index Laws with Negative Indices

- They all apply the same way, but we need to remember a negative sign
- **Multiplication**: $a^m \times a^{-n} = a^{m+(-n)} = a^{m-n}$
- **Division**: $a^m \div a^{-n} = a^{m-(-n)} = a^{m+n}$
- **Power of powers**: $(a^m)^{-n} = a^{m\times(-n)} = a^{-mn}$

Learning Intention

Topic: Scientific Notation

7F	**16 I can convert from scientific notation to a basic numeral.** e.g. Write these numbers as a basic numeral: **a** 4.9×10^3 **b** 3.01×10^{-6}	☐
7F	**17 I can write numbers using scientific notation.** e.g. Write these numbers using scientific notation: **a** 27 000 **b** 0.0000375	☐

What if we divide or multiply by a really big power of 10?

For example, if we multiply 2.99792458 by 100000000

- this is a specific value that holds meaning in physics
- but it's also kinda a pain to work with
- physicists love rounding things, so they would use it as:
    - 300000000
- except they do something extra to make it easier to look at
- seriously, how many zeroes is that, how am I supposed to make sure I don't forget one

Introducing.... scientific notation!

We can represent 100000000 as 10^8!

- That means 300000000 can be: 3×10^8
- This form is called scientific notation

Examples of Scientific Notation

- Earth's population: approx 7.951 billion = 7.951×10^9
- Sun's mass: approx 1.988×10^{30} kg
- Stars in the Milky Way: approx 2×10^{11}

Procedure

- We have a number
  - e.g. $27,000$
- We take its first nonzero digit as the whole part, put the rest after a decimal point
  - e.g. 2.7
- What power of 10 did we have to divide our number by to get this?
  - (hint: we can divide the original number by the decimal number)
  - e.g. $27000 \div 2.7$
- Now we take the two parts and put them together with a times sign (×)
  - 2.7×10^4
- And that's it!

Negative Powers of 10

- Scientific notation also works for very small numbers (numbers with many decimal places)
- Example: the mass of an oxygen molecule is 0.000000000000000000000000053 g, which we write as 5.3×10^{-26}
- Here, we see what power of 10 we had to multiply to get 5.3
  - (count the zeros before and after the decimal point)

Learning Intention

Topic: Scientific Notation with Significant Figures

7G	**18 I can write numbers using scientific notation and rounding to a given number of significant figures.** e.g. Write these numbers in scientific notation using three significant figures: **a** 9 143 000 **b** 0.00032	☐
7G	**19 I can use a calculator to evaluate expressions involving numbers expressed with scientific notation.** e.g. Use a calculator to evaluate $\frac{\sqrt{5.3 \times 10^{-3}}}{8.32 \times 10^{-2}}$ and express your answer using four significant figures.	☐

Significant Figures

- In 1856, the Surveyor General of India, Andrew Waugh, measured Mt. Everest as exactly 29,000 ft
- But he announced it was 29,002 ft so people wouldn't think it was an estimate
- Historians called him "the first person to put two feet on top of Mount Everest"
- How did his change make the number seem more accurate?
  - More formally, it increased the "**significant figures**" in the number
- **Significant figures are the important digits which indicate how accurate a number is**

- If Earth's population is 7.951 billion, are there exactly 7,951,000,000 people?
  - There are 4 significant figures in this number
- We count significant figures starting with the first non-zero digit on the left, e.g.
  - 38041: there are 5 significant figures
  - 6.034: there are 4 significant figures
  - 0.0016: there are 2 significant figures
  - 0.00160: there are 3 significant figures
- "Trailing zeros" (zeros at the end of a number)
  - e.g. 700000m vs. 1.500m
  - They count as significant figures after the decimal point, but not for whole numbers (unless specified)

- In scientific notation, the first significant figure is the whole number to the left of the decimal point, e.g. $\mathbf{7}.951 \times 10^9$

Calculators can be used to work with scientific notation.

- [EXP] or [$\times 10^x$] are common key names on calculators.
- Pressing 2.37 [$\times 10^x$] 5 gives 2.37×10^5.
- 2.37E5 means 2.37×10^5.